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29 Avril
03 Mai

INSTITUT D'ÉTUDES SCIENTIFIQUES DE CARGÈSE

Real and Complex Singularities in Cargese

Goulwen FICHOU
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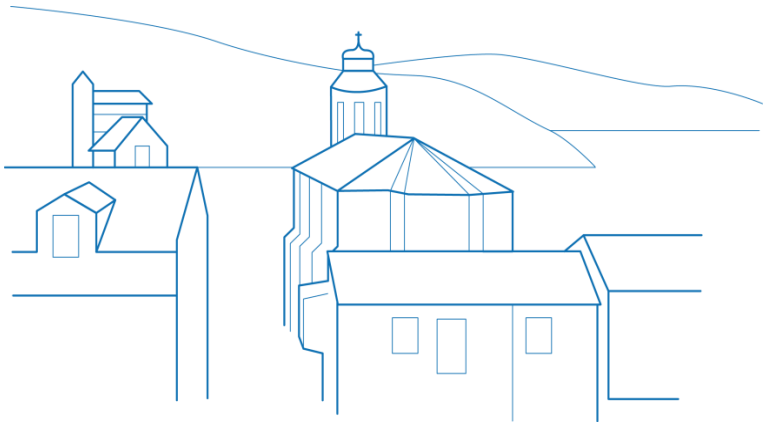

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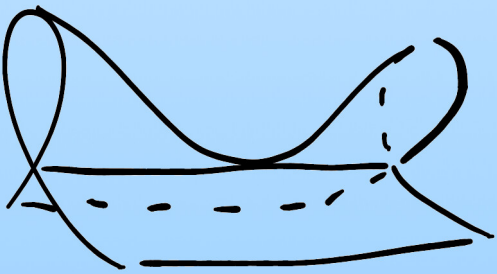
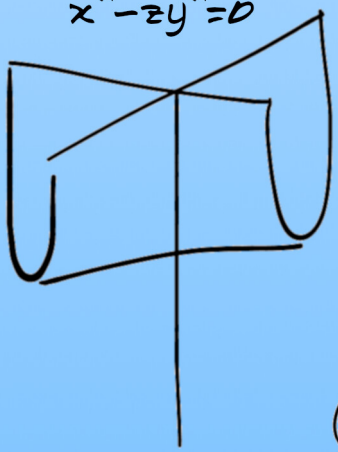
Real and complex singularities in Cargèse

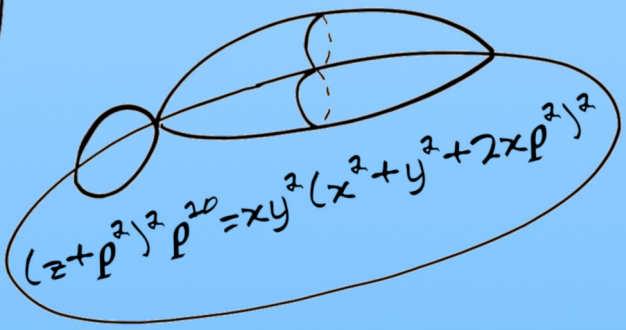
Apr 29–May 3, 2019



$x^3+y^2-z^2x^2=0$

$x^2-zy^2=0$





Participants

André Belotto da Silva	*Aix-Marseille Université*
Edward Bierstone	*University of Toronto*
Marcin Bilski	*Jagiellonian University*
Arnaud Bodin	*Université de Lille*
David Bourqui	*Université de Rennes 1*
Jean-Baptiste Campesato	*University of Toronto*
Georges Comte	*Université Savoie Mont Blanc*
Michel Coste	*Université Rennes 1*
Alexandru Dimca	*Université Nice Sophia Antipolis*
Nicolas Dutertre	*Université d'Angers*
Mohammed Elamrani	*Université d'Angers*
Goulwen Fichou	*Université Rennes 1*
Toshizumi Fukui	*Saitama University*
André Galligo	*Université Côte d'Azur*
Riccardo Ghiloni	*University of Trento*
Victor Goryunov	*University of Liverpool*
Vincent Grandjean	*UFC*
Michel Granger	*Université d'Angers.*
Benoît Guerville-Ballé	*University of Sao Paulo*
Helmut Hamm	*Universitaet Muenster*
Avner Kiro	*Tel Aviv university*
Iwona Krzyzanowska	*University of Gdańsk*
Krzysztof Kurdyka	*Université Savoie Mont Blanc*
Kevin Langlois	*Heinrich Heine University*
Lê Dũng Tráng	*Universite Aix-Marseille*
Olivier Le Gal	*Université Savoie Mont Blanc*
Ann Lemahieu	*Université Côte d'Azur*
Ursula Ludwig	*Universität Duisburg-Essen*
Clint McCrory	*University of Georgia*
Maria Michalska	*Universidade Federal de Ceara*
Aleksandra Nowel	*University of Gdańsk*
Jesus Alberto Palma Marquez	*Universidad Nacional Autónoma de México and Universidad de Valladolid*

AP60 – Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 am	Welcome to AP60		Zbigniew Szafraniec		
9:30 am	Clint McCrory	Ludovic Rifford		Ursula Ludwig	Marcin Bilski
10:00 am			Fernando Sanz Sánchez		
10:30 am	Coffee Break	Coffee Break		Coffee Break	Coffee Break
11:00 am	Armin Rainer	André Belotto da Silva		Claude Sabbah	Guillaume Rond
11:30 am			Fabien Priziac *11:20–12:20*		
12:00 am	Lunch	Lunch		Lunch	Lunch
12:30 am			Lunch		
1:00 pm					
1:30 pm					
2:00 pm					
2:30 pm	Georges Comte	Krzysztof Kurdyka		Alexandru Dimca	Toshizumi Fukui
3:00 pm					
3:30 pm	Coffee Break	Coffee Break		Coffee Break	Coffee Break
4:00 pm	Jean-Philippe Rolin	Maria Pe Pereira		Piotr Pragacz	Laurenţiu Păunescu
4:30 pm					
5:00 pm					

Schedule: apéritif and dinner

A welcome *apéritif* will take place on Monday at 6:30pm.

The conference dinner will take place on Thursday at 7:30pm.

Monomialization of a quasianalytic morphism

André Belotto da Silva
Aix-Marseille Université

I will present a monomialization theorem for mappings in general classes of infinitely differentiable functions that are called quasianalytic (work in collaboration with Edward Bierstone). Examples include Denjoy-Carleman classes (of interest in real analysis), the class of infinitely differentiable functions which are definable in a given polynomially bounded o-minimal structure (in model theory), as well as the classes of real- or complex-analytic functions, and algebraic functions over any field of characteristic zero. The monomialization theorem asserts that mapping in a quasianalytic class can be transformed to mapping whose components are monomials with respect to suitable local coordinates, by sequences of simple modifications of the source and target (local blowings-up and power substitutions in the real cases, in general, and local blowings-up alone in the algebraic or analytic cases). It is not possible, in general, to monomialize by global blowings-up, even in the real analytic case.
The problem of monomialization has been considered a problem in algebraic geometry, and has an extensive literature. The result has previously been proved in the algebraic and analytic cases by D. Cutkosky, using valuation theory. Our point of view is rather that of analysis, and we develop a calculus of derivations tangent to the fibres of a morphism, which is valid for any class satisfying the quasianalytic axioms. Applications of monomialization include new results on the rectilinearization of sub-quasianalytic sets, that were obtained for the class of definable functions by J.-P. Rolin and T. Servi, using model-theoretic techniques.

On algebraic approximation of continuous maps into spheres

Marcin Bilski
Jagiellonian University of Kraków

It is well known that regular functions are too rigid to approximate an arbitrary continuous map from a compact set in $\mathbb{R}^n$ into a sphere. Therefore approximation properties of larger classes of functions have been investigated. One of such classes is the class of regulous functions intensively studied in recent years. It turns out that every continuous map from a compact set in $\mathbb{R}^n$ into a sphere can be approximated by means of functions which are obtained from regulous functions by sign changing on some subsets of their domains.

TBA

Georges Comte
Université Savoie Mont Blanc

TBA

Generators of the Jacobian syzygy module, and rational cuspidal curves

Alexandru Dimca
Université Nice Sophia Antipolis

We give upper bounds for the number and degrees of generators of the module of Jacobian syzygies of a reduced plane curve. Then we relate these numbers to rational (nearly) cuspidal curves, and to curves realizing the maximum Tjurina number in a well known du Plessis-Wall inequality.

On bifurcation model for several nonlinear problems

Toshizumi Fukui
Saitama University

We are going to talk about an application of singularity theory to the bifurcation of solutions to several non-linear equations.
We present a bifurcation model determined by the initial non-linear term of the non-linear equation in the case that the linear term has the eigenspace of finite dimension. We talk how this model works several examples, e.g. Dirichlet problem on a square.
This is a joint work with Qiang Li and Donghe Pei.

Arc-Nash functions

Krzysztof Kurdyka
Université Savoie Mont Blanc

We study a concept of an arc-Nash function i.e., when composed with a Nash arc it remains a Nash function of one variable. The main result states that such a function (defined on a semialgebraic arc-symmetric set) is semialgebraic and arc-analytic. In particular it is blow-Nash, i.e. it becomes a Nash function when composed with a suitable finite composition of blowing-ups.
This is in a strong contrast with the fact that there are arc-analytic functions which are not subanalytic, even not continuous (an exemple due to Bierstone, Milman and Parusiński). Arc-Nash functions give rise to a natural class of sheaves for which Cartan's theorems A and B hold.
Joint work with W. Kucharz.

An Extension of a Theorem by Cheeger and Müller to Spaces with Isolated Conical Singularities

Ursula Ludwig
University Duisburg-Essen

An important comparison theorem in global analysis is the comparison of analytic and topological torsion for smooth compact manifolds equipped with a unitary flat vector bundle. It has been conjectured by Ray and Singer and has been independently proved by Cheeger and Müller in the 70ies. Bismut and Zhang combined the Witten deformation and local index techniques to generalise the result of Cheeger and Müller to arbitrary at vector bundles with arbitrary Hermitian metrics.
The aim of this talk is to present an extension of the Cheeger-Müller theorem to spaces with isolated conical singularities by generalising the proof of Bismut and Zhang to the singular setting. In the first part of the talk, I will recall the classical Cheeger-Müller theorem for smooth manifolds.

Hyperresolutions: complex and real

Clint McCrory
University of Georgia

A hyperresolution of an algebraic variety X is a diagram of smooth varieties over X that in some sense captures the geometry of X.
I will survey the constructions and applications of hyperresolutions, including the approaches of Deligne, Gillet-Soulé, Guillen-Navarro Aznar, and Peters-Steenbrink, as well as my work with Adam Parusiński on the weight complex for real algebraic varieties.

Zariski Equisingularity and Lipschitz stratification of a family of surface singularities

Laurenţiu Păunescu
University of Sydney

(joint with A. Parusiński)
We consider the (generic) Zariski equisingular families of surface (not necessarily isolated) singularities in $\mathbb{K}^3$, $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$.
We show that a natural stratification of such a family given by the singular set and the generic family of polar curves provides a Lipschitz stratification in the sense of Mostowski.
In particular, such families are bi-Lipschitz trivial by a trivialization obtained by the integration of a Lipschitz vector field.

Moderatly discontinuous homology

Maria Pe Pereira
Complutense University of Madrid

I will introduce a new notion of metric homology, invariant up to bilipschitz equivalence. It applies in particular to study both the inner and outer metric of algebraic germs. I will give the basic definitions and give some examples. This is a joint work with J. Fernández de Bobadilla, S. Heinze and E. Sampaio.

Order of tangency between manifolds

Piotr Pragacz
IMPAN, Warsaw

We study the order of tangency between two manifolds of same dimension and give that notion three quite different interpretations: by Taylor series, by a mini-max procedure and by Grassmannians.
Related aspects of the order of tangency, e.g., regular separation exponents, are also discussed.
This is a joint work with Wojciech Domitrz and Piotr Mormul.

Quotients and invariants of $\mathcal{AS}$-sets equipped with a finite group action

Fabien Priziac
Aix-Marseille Université

Let G be a finite group. The real geometric quotient of a compact arc-symmetric set (e.g. a compact algebraic set) equipped with a free action of G is a compact arc-symmetric set. We will see how this fact allows to construct invariants for $\mathcal{AS}$-sets (in particular for affine algebraic varieties) equipped with a biregular action of G, with respect to equivariant homeomorphisms with $\mathcal{AS}$-graphs, including additive invariants with values in $\mathbb{Z}$.

Polynomial equations with smooth coefficients

Armin Rainer
University of Vienna

The roots of a monic polynomial of degree n with smooth complex-valued coefficients form a multi-valued function which admits smooth parameterizations locally near points, where all roots are distinct.
In this talk I will pursue the question how regular the roots can be parameterized near their contact points.
This question arises naturally in a wide array of mathematical problems and is of particular interest if the coefficients are assumed to be just C^∞ or of finite differentiability.
I will survey the recent developments in this field.
The focus will be on the optimal Sobolev regularity of the roots and on their selections of bounded variation in the multiparameter case when discontinuities are unavoidable due to monodromy phenomena.
The talk is based on joint work with Adam Parusiński.

Singularities in Sub-Riemannian Geometry

Ludovic Rifford
Université Nice Sophia Antipolis

The sub-Riemannian geometry is concerned with the geometric study of metric spaces where the points can only be connected through curves, called horizontal, which have to be tangent to a given totally nonholonomic distribution at any point. In many cases, the set of horizontal curves between two points admit singularities, a certain type of curve, called singular curve, along which the distribution has a degenerate behavior. One of the main open problem in sub-Riemannian geometry deals with the set of points that can be attained through such curves starting from a given point. The Sard conjecture states that this set should have Lebesgue measure zero and even Hausdorff dimension 1 in an ambiant space of dimension 3. We are going to present a proof of the latter result in the case of analytic sub-Riemannian structure and discuss an application to the regularity of minimizing geodesics. The techniques rely on symplectic geometry and resolution of singularities.
This is a joint work with André Belotto, Alessio Figalli and Adam Parusiński.

Orbits of Dulac maps in the neighborhood of a singular point

Jean-Philippe Rolin
Université de Bourgogne

Given a continuous or discrete dynamical system, it is natural to ask to what extent it can be identified by some of its orbits. The sytems considered in this talk are generated by "Dulac maps", which appear in various works dedicated to Hilbert's sixteen problem. A Dulac map is analytic on some open interval $(0, d)$, and admits a non-trivial power-logarithmic asymptotic expansion at 0.
We show that a convenient asymptotic analysis of the epsilon-neighborhood of one its orbits allows to determine the formal class of such a system. The proof involves various aspects of iteration theory, as well as several manipulations of transseries.
Joint work with P. Mardesic, M. Resman and V. Zupanovic.

Local holomorphic equivalence of real algebraic sets

Guillaume Rond
Aix-Marseille Université

Given two germs of real algebraic or analytic manifolds M and M' in the affine complex space $\mathbb{C}^n$, a general problem is to determine when these exists a germ of a biholomorphic map $h : \mathbb{C}^n \longrightarrow \mathbb{C}^n$ sending one onto the other. We will review this problem, emphasizing on the question of the existence of such a biholomorphism germ h under the assumption of the existence of a germ of a formal biholomorphism $\widehat{h}$ such that $\widehat{h}(M) = M'$. Finally we will present an example, based on the on study of the generating sequence of some walk restricted to the quarter plane, giving (almost) a negative answer to a question of Baouendi, Rothschild and Zaitsev.

The Riemann-Hilbert correspondence for irregular singularities

Claude Sabbah
École Polytechnique

In the talk, I will survey some recent results of d'Agnolo, Kashiwara, Schapira, T. Mochizuki and Kuwagaki on the Riemann-Hilbert correspondence for holonomic complex differential systems with possibly irregular singularities. I will focus on a recent result due to T. Mochizuki, which exploits the link between complex and subanalytic geometries produced by such systems of differential equations.

Alternative interlacing/Hardy-compatibility of solutions of ODEs with definable coefficients

Fernando Sanz Sánchez
University of Valladolid

Framed in the qualitative study of the dynamics of three-dimensional real vector fields, and concerning specially the relative behavior of trajectories, there exists an heuristic principle that states that given a pair of trajectories, either they have infinite twisting (interlaced) or they reveal good finiteness properties.
In this talk we present a result (obtained in collaboration with M. Matusinski and O. LeGal) that provides another manifestation of this principle, with a precise meaning of "finiteness property", in a general context of vector fields definable in a polynomially bounded o-minimal structure. More precisely, written such a vector field as a system of ODEs of the form

$$\frac{dY}{dx} = F(x, Y), \text{ where } Y = (y_1, y_2), \ F \text{ definable of class } C^1 \qquad (1)$$

and given solutions $\gamma, \delta : (0, \epsilon) \to \mathbb{R}^2$ of (1) with exponentially flat contact and such that γ has the *regular separation* property with respect to the structure, then either γ, δ are interlaced or the ring of functions of the form $x \mapsto h(x, \gamma(x), \delta(x))$, where h is definable, is a Hardy field.
Applied to analytic vector fields, this result refines the alternative *interlacing/separation by a projection*, established by Cano, Moussu and the speaker, for certain integral pencils of non-oscillating trajectories. Notably, those pencils with transcendental formal axis.

On bifurcations of cusps

Zbigniew Szafraniec
University of Gdańsk

Let $f : (\mathbb{R}\times\mathbb{R}^2, 0) \to (\mathbb{R}^2, 0)$ be an analytic mapping having a critical point at the origin. There is the corresponding one-parameter family of mappings $f_t = f(t, \cdot) : \mathbb{R}^2 \to \mathbb{R}^2$.
There will be presented effective algebraic methods of computing the number of cusps of f_t, where $0 < |t| \ll 1$, emanating from the origin and having a positive/negative cusp degree.

Real and complex singularities in Cargèse

Conference in Honour of the 60th birthday of

Adam Parusiński

Apr 29–May 3, 2019

SPEAKERS

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Yacoub Moine

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